# The Day Before Pure 1 Livestream 2026

Questions written by the very talented A-Level Maths and Further Maths student, Gavin Arora, and co-edited with Mr Bicen

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# Question 1: Differentiation / Proof by Contradiction

(a) Show that the equation

$$y = 3x - 2\sin^2(x)$$

has a gradient given by

$$\frac{dy}{dx} = 3 - 2\sin(2x)$$

**(3)**

(b) Hence, prove by contradiction that the curve $y = 3x - 2\sin^2(x)$ has no real stationary points.

**(3)**

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## Question 2: Modulus Graphs

The function $f$ is defined by $f(x) = |3x - 12|$.

(a) Find the coordinates of the vertex of the graph $y = f(x)$.

**(2)**

The function $g$ is defined by $g(x) = f(x - a) + 2a$, where $a \in \mathbb{R}$.

(b) By using your answer to part (a), determine the coordinates of the vertex of the graph $y = g(x)$, giving your answer in terms of $a$.

**(2)**

(c) Hence, given that the graph of $y = \frac{x}{2} + 4$ intersects $y = g(x)$ exactly once, determine the value of $a$.

**(3)**

## Question 3: Small Angle Approximations / Binomial Expansion

(a) Show that, for small values of $x$, measured in radians,

$$\frac{2\tan 3x + \cos 4x - 1}{2x} \approx 3 - 4x$$ **(3)**

(b) Hence, given that $x$ is small, find in ascending powers of $x$ the first three terms of the binomial expansion of

$$\left(\frac{4\tan 3x + 2\cos 4x - 2}{2x} + 2\right)^{\frac{1}{3}}$$

giving each term in its simplest form. **(4)**

# Question 4: Modelling with Logarithms

A quant analyst is modelling the relationship between the price of a commodity per kg, £$P$, and its daily trading volume, $V$ (in **millions** of units).

The analyst decides to model this relationship using the equation

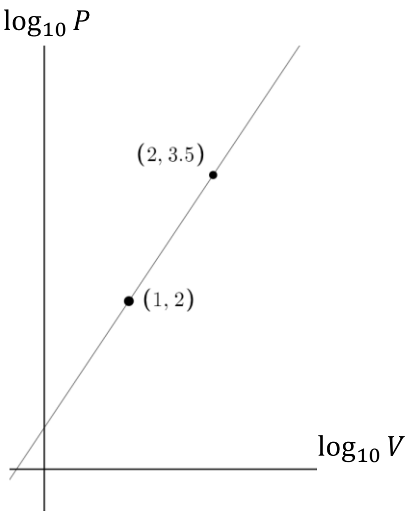
$$P = aV^k$$

where $a$ and $k$ are constants.

(a) (i) Show that the relationship between $P$ and $V$ can be expressed as

$$\log_{10}(P) = k\log_{10}(V) + \log_{10}(a)$$

**(2)**




The analyst plots a graph of $\log_{10}(P)$ against $\log_{10}(V)$.

The resulting straight line passes through the points $(1, 2)$ and $(2, 3.5)$.

(a) (ii) Find the values of $k$ and $a$.

**(3)**

(b) The analyst is assessing which of two commodities has a higher value.
Commodity A's price is estimated to be £2.57 per kg.
Commodity B's daily trading volume is 890,000 units.
According to the model, which commodity has a higher value?

**(2)**



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# Question 5: Series

An athlete, Person A, begins a strength-training program. Their initial bench press maximum is 60 kg.

In Week 1, their new maximum is 65 kg.
In Week 2, their new maximum is 69 kg.
In Week 3, their new maximum is 72.2 kg.

The **weekly increases in Person A's bench press maximum** are modelled as a geometric series.

(a) Show that the common ratio $r$ for this model is 0.8.

**(3)**

(b) Find the total increase in Person A's bench press after 8 weeks, giving your answer to 1 decimal place.

**(2)**

(c) Using the model, find the theoretical maximum mass Person A can ever bench press.

**(3)**

Another athlete, Person B, starts training at the same time as Person A.

Their **bench press maximums each week** follow an arithmetic progression, $u_1, u_2, u_3, \ldots$ where $u_1 = 55$ kg and the common difference is $d$ kg.

Given that Person B's bench press maximum exceeds Person A's theoretical maximum in Week 16,

(d) Find the range of possible values of $d$.

**(3)**

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(a) By rewriting $\tan(3\theta)$ as $\tan(2\theta + \theta)$, show that

$$\tan(3\theta) \equiv \frac{3\tan(\theta) - \tan^3(\theta)}{1 - 3\tan^2(\theta)}$$

**(5)**

(b) Hence, solve, for $0 \leq x < 180°$, the equation

$$\frac{3\tan(x) - \tan^3(x)}{1 - 3\tan^2(x)} = \sqrt{3}$$

**(4)**

(c) Determine the **number of solutions** for

$$\frac{3\tan\left(\frac{2x}{3}\right) - \tan^3\left(\frac{2x}{3}\right)}{1 - 3\tan^2\left(\frac{2x}{3}\right)} = \sqrt{3}$$

where $-900 \leq x \leq 900°$, explaining your reasoning clearly.

**(3)**

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The Large Hadron Collider (LHC) is the world's largest and most powerful particle accelerator, built by CERN. It smashes subatomic particles (hadrons) together at near-light speed, allowing scientists to study fundamental physics and the structure of matter.

In a specific experimental sector of the LHC, the position $(x, y)$, in **kilometres**, of a proton relative to a monitoring station at the origin $O$ is modelled by the parametric equations

$$x = 4\cos(t) + 4, \qquad y = 4\sin(t) - 3$$

where $t$ is the time in seconds.

(a) Show that the motion of the proton is governed by the differential equation

$$\frac{dy}{dx} = \frac{4 - x}{y + 3}$$

**(3)**

(b) By solving the differential equation, show that the motion of the proton follows a **circular** path.

**(5)**

(c) Hence, calculate the total distance that the proton travels in one full revolution around the LHC, giving your answer in terms of $\pi$.

**(1)**

(d) Given that the actual Large Hadron Collider has a circumference of approximately 27km, comment on the suitability of the model.

**(1)**




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**Figure 1** shows the curve $C$ with the equation

$$y = \frac{2}{1 + 4x^2}$$

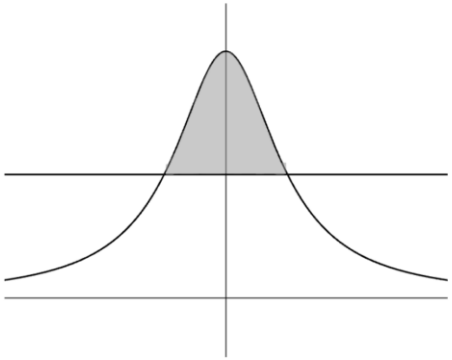

**Figure 1**

The finite region $S$, shown shaded, is bounded by the curve $C$ and the horizontal line $y = 1$.

(a) By using the substitution $2x = \tan(\theta)$, show that the finite region $S$ has an area of $\left(\frac{\pi}{2} - 1\right)$ units$^2$.

**(7)**

An architect uses the region $S$ to model part of a roller coaster at a local theme park.

The region $S$ is used to represent the side profile of the roller coaster.

The architect wants to hide the support beams below the roller coaster track, by covering the entire side profile of the roller coaster with panelling.

(b) Given that the architect has $514 \text{ m}^2$ of panelling available to use, determine, to the nearest metre, the greatest possible height of the roller coaster.

**(3)**



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# Question 9: Modelling with the Harmonic Identity

The Moon's gravitational pull is the primary cause of Earth's tides.

As the Moon orbits the Earth, its gravity pulls on the oceans to create a "bulge" of water. Because the Earth rotates, a specific location passes through these bulges, typically resulting in two high tides and two low tides every 24 hours.

Since this pattern repeats, we can use trigonometric functions to model the changing tide heights over time.

The height of the tide, $H$ metres, at a particular port is modelled by the equation

$$H(t) = 5 + 3\sin\left(\frac{\pi t}{6}\right) + 4\cos\left(\frac{\pi t}{6}\right), \qquad 0 \leq t \leq 24$$

where $t$ is the time in hours after midnight.

(a) Express $H(t)$ in the form $5 + R\cos\left(\frac{\pi t}{6} - \alpha\right)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

**(4)**

(b) Find the time of the first high tide after midnight, giving your answer to the nearest minute.

**(3)**

Due to the melting of polar ice caps, global sea levels have risen. Local scientists at the port have observed that both the maximum and minimum tidal heights have increased by 0.4 m, while the tidal frequency remains unchanged.

(c) Suggest a new value for the constant term in the model $H(t)$.

**(1)**





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