The Day Before CP1 Livestream 2026

Questions written by the very talented A-Level Maths and Further Maths student, Gavin Arora, and co-edited with Mr Bicen

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£15 tickets for 'in between' CP1 and CP2 session, Saturday 16th May – see video description for link

Question 1: Complex Numbers

The complex number $z = x + iy$ satisfies the equation

$$\frac{zz^* + k}{z} = 6i + 2$$

where k is a real constant.

(a) Given that this equation has exactly one root, determine the value of k.

(5)

(b) Hence, show that $z = 1 - 3i$.

(3)

The cubic equation $f(x) = x^3 + px^2 + qx + r$, where p, q and r are real constants has roots α, β, γ

(c) Given that $\alpha = z, \beta = z^*$ and $\gamma = 4$, determine the values of p, q and r **(3)**

(d) Using the relationships between roots and coefficients, determine the exact value of $\alpha^2 + \beta^2 + \gamma^2$ **(3)**

Question 2: De Moivre's theorem

(a) Given that $z = \cos(\theta) + i\sin(\theta)$, prove by induction that for all positive integers n,

$$z^n = \cos(n\theta) + i\sin(n\theta)$$

(6)

(b) Using De Moivre's theorem, show that

$$\cos(6\theta) \equiv 32\cos^6(\theta) - 48\cos^4(\theta) + 18\cos^2(\theta) - 1$$

(5)

(c) Hence, find the six distinct solutions of the equation

$$32x^6 - 48x^4 + 18x^2 - \frac{3}{2} = 0$$

giving your answers to 3 decimal places.

(6)

Question 3: Matrices

A nutritionist is monitoring the micronutrient levels of Patient Alpha, measured in milligrams. Initially, the patient has 1000 mg of Iron, 1500 mg of Magnesium and 2500 mg of Zinc.

After one month of taking Supplement A, the Iron level increased by $x\%$, the Magnesium level increased by $y\%$ and the Zinc level increased by $z\%$. The nutritionist also noted that

- The total combined increase across all three micronutrients was 155 mg
- The increase in Magnesium was 35 mg less than the combined increase of Iron and Zinc
- The increase in Magnesium was 40 mg more than the increase in Iron

(a) By forming and solving a 3×3 matrix, find the values of x, y and z.

(6)

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Patient Beta, who has the same initial micronutrient levels as Patient Alpha, thinks that Supplement A is too expensive and is looking for a cheaper alternative.

The nutritionist recommends Supplement B to Patient Beta, which only affects **Iron** and **Magnesium** levels.

After one month of taking Supplement B, the Iron level increased by $p\%$ and the Magnesium level increased by $q\%$. The nutritionist also noted that

- The combined increase of Iron and Magnesium was 80 mg
- The increase in Magnesium was 20 mg more than the increase in Iron

(b) Use an algebraic method to find the values of p and q.

(2)

The effectiveness of a supplement is determined by considering the average percentage increase it results in across the nutrients it affects.

(c) By using your answers to parts (a) and (b), determine whether Supplement A or Supplement B is more effective.

(2)

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Question 4: Polar Coordinates

Figure 6 shows a sketch of part of the curve C with polar equation

$$r = 1 + \tan(\theta), \qquad 0 \leq \theta < \frac{\pi}{2}$$

The tangent to the curve C at the point P is perpendicular to the initial line.

(a) Find the polar coordinates of the point P.

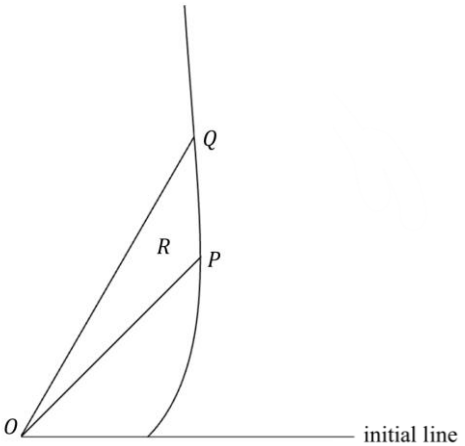

Figure 6

(5)

The point Q lies on the curve C. The region R is bounded by OP, OQ and the curve C, as shown in **Figure 6**.

(b) Given that $r = 1 + \sqrt{3}$ at the point Q, find the exact area of R, giving your answer in the form

$$\frac{1}{2}(\ln p + \sqrt{q} + r)$$

where p, q and r are integers to be found.

(7)

Question 5: Vectors

A helicopter is hovering at a fixed point $H(100, 200, 1500)$. In this model, the origin O is on horizontal ground. The unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ point due East and due North respectively, and the unit vector $\boldsymbol{k}$ points vertically upwards. All coordinates and distances are given in metres.

A skydiver jumps from the helicopter at $t = 0$ seconds. For the first 10 seconds of the jump, her position is modelled by the line

$$r_1 = \begin{pmatrix} 100 \\ 200 \\ 1500 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -50 \end{pmatrix}, \quad 0 \leq t \leq 10$$

At $t = 10$ seconds, the skydiver opens her parachute. Her motion is now modelled by a second line r_2. She now moves at $(\boldsymbol{i} + 4\boldsymbol{j} - 16\boldsymbol{k})$ metres per second.

The skydiver is modelled as a particle.
The ground is modelled as a horizontal plane at $z = 0$.
The effect of air resistance is ignored.

(a) Find the vector equation of the line r_2 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{d}$, where s is the time in seconds after the parachute opened.

(2)

(b) Find the time taken for the skydiver to reach the ground from the moment that she jumped from the helicopter.

(2)

A drone is hovering at $D(151, 370, 520)$ to capture footage for a skydiving documentary.

(c) Calculate the shortest distance between the drone and the skydiver's path r_2.

(5)

(d) By considering your answer to part (c), evaluate the suitability of modelling the skydiver as a particle.

(1)

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Question 6: Volumes of Revolution

(a) By using the substitution $u = x^2 + 1$, show that

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx = \frac{a - \sqrt{a}}{b}$$

where a and b are integers to be found.

(3)

Figure 1 shows the curve C, which is defined by the equation

$$y = x\sqrt{\text{arsinh}(x)}, \quad 0 \leq x \leq 1$$

A local jeweller manufactures and sells gold earrings.

A **single** earring can be modelled by a solid of revolution of curve C 2π radians about the x-axis, with its volume being measured in cm^3.

The jeweller purchases gold from a supplier who sells 1cm^3 of gold for £2087.70.

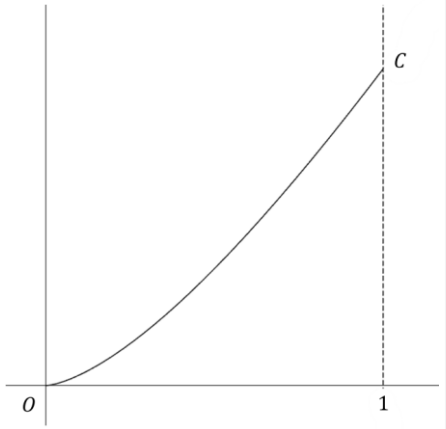

Figure 1

(b) By using your answer to part (a), use the model to find the cost, in £, of manufacturing a **pair** of earrings, giving your answer to the nearest integer.

(6)

Question 7: Differential Equations

A truck's suspension system is modelled as a mass-spring system. The **vertical** displacement y metres of the truck above its equilibrium position at time t seconds is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 4y = \omega \cos(4t)$$

where ω is a positive constant.

Initially, the truck is at rest and is at its equilibrium position. At $t = \frac{\pi}{4}$, the **vertical** velocity of the truck is $-0.5\ \text{ms}^{-1}$.

(a) Find a general solution for the differential equation, giving your answer in terms of ω.

(6)

(b) Hence, determine a particular solution for the differential equation, showing that $\omega = 3$.

(4)

(c) Using your answer from part (b), find the minimum vertical displacement of the truck between $1 < t < 2$.

(4)